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RADIAL ANGULAR TEMPERATURE DISTRIBUTIONS FOR NONISOTHERMAL TWO-PHASE FILTRATION OF OIL AND WATER

R. A. Valiullin, R. F. Sharafutdinov,
A. A. Sadretdinov, and A. S. Bochkov

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The distributions of phase saturations, pressure, and temperature in a porous medium of nonuniform permeability are studied by numerical modeling of nonisothermal two-phase filtration of oil and water with the Joule–Thomson effect and adiabatic effect taken into account. It is shown that the presence of nonuniformity in the near-well zone of the reservoir results in nonmonotonic angular and radial distributions of temperature and phase saturations. During oil and water filtration, there is transition from negative to positive temperature anomalies or vice versa, depending on the ratio of the reservoir permeabilities and the presence of a segment on which the angular temperature distribution in the well is nonuniform.

Key words: *temperature, Joule–Thomson effect, adiabatic effect, radial angular nonuniformity of permeability distribution, well, reservoir, two-phase filtration.*

During nonisothermal filtration of oil and water, the Joule–Thomson effect and the adiabatic effect influence the temperature distribution in the reservoir [1, 2]. Temperature variation laws underlies a thermal method for studying wells and reservoirs [1–3].

During formation drilling, the permeability in the bottomhole zone changes. The radius of the zone of nonuniform permeability distribution can be a few centimeters to tens meters. During well operation, the bottomhole zone is polluted by asphaltic components, resulting in a decrease in its permeability [4]. The nonuniform permeability of the bottomhole zone can be caused by reservoir-scale fracturing, hydraulic fracture of formations, etc.

The well-known mathematical models for the temperature field with the Joule–Thomson effect and adiabatic effect taken into account were developed primarily for the case of one-dimensional oil and water filtration. These models ignore nonuniformity in the angular distribution of permeability in the bottomhole zones [1–3, 5–7].

In the present paper, a non-stationary temperature field in a reservoir of nonuniform permeability during oil and water filtration is studied numerically with the Joule–Thomson effect and adiabatic effect taken into account. The phase motion is described by the Darcy law, and the mass diffusion transfer and the liquid phase transfer due to a jump of the capillary pressure in the phases are ignored.

Ignoring thermal losses, we formulate a mathematical model for the radial angular temperature distribution [3] due to the Joule–Thomson effect and the adiabatic effect for oil and water filtration in a nonuniform reservoir. Let the reservoir have nonuniform permeability in the region $\{\omega\colon r_{1,\text{oil}} \leqslant r \leqslant r_{2,\text{oil}},\ -\alpha_{\text{oil}} \leqslant \alpha \leqslant \alpha_{\text{oil}}\}$ (Fig. 1). As a first approximation, we can distinguish three phases (phase 0 refers to the porous skeleton, phase 1 to oil, and phase 2 to water) and two components participating in the heat- and mass-transfer processes (component 1 refers to oil and component 2 to water).

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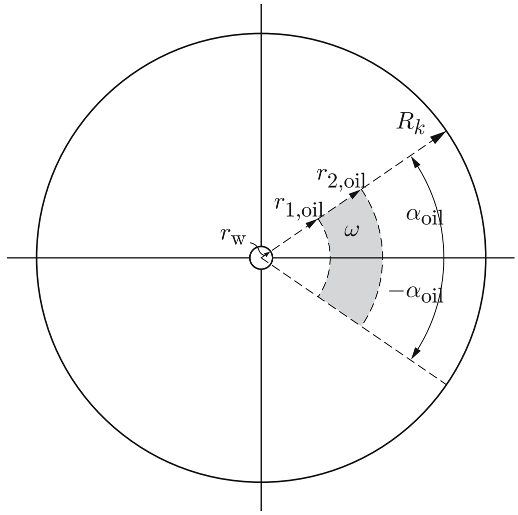

Fig. 1. Geometry of the problem (the dashed region is the region of nonuniform permeability distribution).

In the two-dimensional (r, α) case, the mathematical models have the following form:

— the equations of mass conservation for the phases are

$$m\,\frac{\partial S_1}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{Kk_1(S_1)}{\mu_1}\,\frac{\partial P}{\partial r}\Big) + \frac{1}{r^2}\,\frac{\partial}{\partial \alpha}\Big(\frac{Kk_1(S_1)}{\mu_1}\,\frac{\partial P}{\partial \alpha}\Big),$$
$$m\,\frac{\partial S_2}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{Kk_2(S_2)}{\mu_2}\,\frac{\partial P}{\partial r}\Big) + \frac{1}{r^2}\,\frac{\partial}{\partial \alpha}\Big(\frac{Kk_2(S_2)}{\mu_2}\,\frac{\partial P}{\partial \alpha}\Big); \tag{1}$$

— the equation of heat influx is

$$\frac{\partial}{\partial t}\,[m(\rho_1 c_1 S_1 + \rho_2 c_2 S_2)T + (1-m)\rho_0 c_0 T] + \frac{1}{r}\,\frac{\partial}{\partial r}\,[r(\rho_1 c_1 v_1 + \rho_2 c_2 v_2)T]$$
$$+ \frac{1}{r}\,\frac{\partial}{\partial \alpha}\,[(\rho_1 c_1 v_1 + \rho_2 c_2 v_2)T] = \frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\lambda_r\,\frac{\partial T}{\partial r}\Big) + \frac{1}{r^2}\,\frac{\partial}{\partial \alpha}\Big(\lambda_\alpha\,\frac{\partial T}{\partial \alpha}\Big)$$
$$+ m(\rho_1 c_1 S_1 \eta_1 + \rho_2 c_2 S_2 \eta_2)\,\frac{\partial P}{\partial t} + (\varepsilon_1 \rho_1 c_1 v_1 + \varepsilon_2 \rho_2 c_2 v_2)\,\frac{\partial P}{\partial r}$$
$$+ \frac{1}{r}\,(\varepsilon_1 \rho_1 c_1 v_1 + \varepsilon_2 \rho_2 c_2 v_2)\,\frac{\partial P}{\partial \alpha}. \tag{2}$$

Here

$$S_1 + S_2 = 1,$$

S_i are the phase saturations, T is the temperature, P is the pressure, v_i are the phase velocities, c_i are the heat capacities of the phases, c_0 is the heat capacity of the rock skeleton, ρ_i are the densities of the phases, K and k_i are the absolute and phase permeabilities, m is the porosity, μ_i is the viscosity, ε_i is the Joule–Thomson coefficient, η_i is the adiabatic coefficient, and λ_r and λ_α are the radial and angular thermal conductivities.

The initial and boundary conditions are written as follows:

— the initial conditions are

$$P(r,\alpha)\Big|_{t=0} = P_{\text{res}}, \quad S_1(r,\alpha)\Big|_{t=0} = S_0, \quad T(r,\alpha)\Big|_{t=0} = T_0$$
$$\text{at} \qquad 0 \leqslant r \leqslant R_k, \quad 0 \leqslant \alpha \leqslant 2\pi;$$

— the boundary conditions are

$$P(r_{\rm w},\alpha,t)=P_{\rm w},\quad P(R_k,\alpha,t)=P_{\rm res},\quad S_2(R_k,\alpha,t)=1,\quad T(R_k,\alpha)=T_0$$

$$\text{at}\qquad 0\leqslant\alpha\leqslant 2\pi,\quad t>0.$$

Here $P_{\rm w}$ is the well pressure, $P_{\rm res}$ is the reservoir pressure, S_0 is the initial oil saturation, T_0 is the reservoir temperature, $r_{\rm w}$ is the well radius, and R_k is the radius of the external boundary.

For the phase permeabilities, the following relations are adopted:

$$k_1=\Big(\frac{S_1-S_1^0}{1-S_1^0}\Big)^3,\qquad k_2=\Big(\frac{S_2-S_2^0}{1-S_2^0}\Big)^{2.5}$$

(S_1^0 and S_2^0 are the residual oil and water saturations, respectively). The thermal parameters of the phases c_i, ε_i, η_i, λ_r, and λ_α are considered constant and are determined from tables [5] for the reservoir-averaged pressure $P=P_0$ and temperature $T=T_0$.

To discretize the basic equations (1) and (2), we use the control volume method. The problem is symmetric about the angle α for the value $\alpha=0$. During integration, the average quantities refer to the node r_i, α_j, t_{n+1}. After discretization (1), we obtain

$$\int\limits_{j-1/2}^{j+1/2}\int\limits_{i-1/2}^{i+1/2}\int\limits_{n}^{n+1}\frac{1}{r}\frac{\partial}{\partial r}\Big(r\Big(\frac{Kk_1(S)}{\mu_1}+\frac{Kk_2(S)}{\mu_2}\Big)\frac{\partial P}{\partial r}\Big)\,dt\,r\,dr\,d\alpha$$

$$+\int\limits_{j-1/2}^{j+1/2}\int\limits_{i-1/2}^{i+1/2}\int\limits_{n}^{n+1}\frac{1}{r^2}\frac{\partial}{\partial\alpha}\Big(\Big(\frac{Kk_1(S)}{\mu_1}+\frac{Kk_2(S)}{\mu_2}\Big)\frac{\partial P}{\partial\alpha}\Big)\,dt\,r\,dr\,d\alpha=0.$$

This equation is brought to the form

$$a_{ij}P_{ij-1}^{n+1}+b_{ij}P_{i-1j}^{n+1}+c_{ij}P_{ij}^{n+1}d_{ij}P_{ij+1}^{n+1}+e_{ij}P_{i+1j}^{n+1}=0,$$

where

$$a_{ij}=(K^*)_{ij-1/2}^{n+1}\frac{1}{\Delta\alpha}\,s_r,\quad b_{ij}=r_{i-1/2}(K^*)_{i-(1/2)j}^{n+1}\frac{1}{\Delta r_i^w}\,s_\alpha,$$

$$c_{ij}=-r_{i+1/2}(K^*)_{i+(1/2)j}^{n+1}\frac{1}{\Delta r_i^e}\,s_\alpha-r_{i-1/2}(K^*)_{i-(1/2)j}^{n+1}\frac{1}{\Delta r_i^w}\,s_\alpha$$

$$-(K^*)_{ij+1/2}^{n+1}\frac{1}{\Delta\alpha}\,s_r-(K^*)_{ij-1/2}^{n+1}\frac{1}{\Delta\alpha}\,s_r,$$

$$d_{ij}=(K^*)_{ij+1/2}^{n+1}\frac{1}{\Delta\alpha}\,s_r,\quad e_{ij}=r_{i+1/2}(K^*)_{i+(1/2)j}^{n+1}\frac{1}{\Delta r_i^e}\,s_\alpha,\quad K^*=\frac{Kk_1(S)}{\mu_1}+\frac{Kk_2(S)}{\mu_2},$$

and s_r, Δr_i^w, Δr_i^e, and s_α are parameters of the computation grid.

To solve the system of equations for the pressure, we use the successive point overrelaxation method:

$$P_{ij}^{(\nu+1)}=(1-\omega)P_{ij}^{(\nu)}-\omega(a_{ij}P_{ij-1}^{(\nu+1)}+b_{ij}P_{i-1j}^{(\nu+1)}+d_{ij}P_{ij+1}^{(\nu)}+e_{ij}P_{i+1j}^{(\nu)})/c_{ij}.$$

The oil saturation in a new time layer is calculated from the known pressure using the explicit formula

$$S_1^{n+1}=S_1^n+\Big(r_{i+1/2}(K_1^*)_{i+(1/2)j}^{n+1}\frac{P_{i+1j}^{n+1}-P_{ij}^{n+1}}{\Delta r_i^e}-r_{i-1/2}(K_1^*)_{i-(1/2)j}^{n+1}\frac{P_{ij}^{n+1}-P_{i-1j}^{n+1}}{\Delta r_i^w}\Big)\frac{\Delta t}{m_{ij}V_{ij}}\,s_\alpha$$

$$+\Big((K_1^*)_{ij+1/2}^{n+1}\frac{P_{ij+1}^{n+1}-P_{ij}^{n+1}}{\Delta\alpha}-(K_1^*)_{ij-1/2}^{n+1}\frac{P_{ij}^{n+1}-P_{ij-1}^{n+1}}{\Delta\alpha}\Big)\frac{\Delta t}{m_{ij}V_{ij}}\,s_r.$$

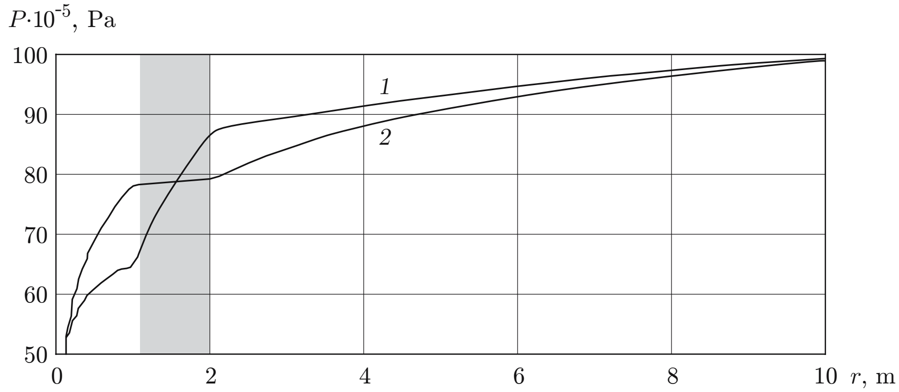

Fig. 2. Reservoir pressure distribution for $k_{\rm oil} = 0.1K$ (1) and $10K$ (2); the dashed region is the region of nonuniform permeability.

After discretization, the heat influx equation becomes

$$\{m[(\rho_1 c_1 S_1 + \rho_2 c_2 S_2)T]_{ij}^{n+1} + (1-m)(\rho_0 c_0 T)_{ij}^{n+1}$$

$$- m[(\rho_1 c_1 S_1 + \rho_2 c_2 S_2)T]_{ij}^{n} - (1-m)(\rho_0 c_0 T)_{ij}^{n}\}V_{ij}$$

$$+ \{[r(\rho_1 c_1 v_1 + \rho_2 c_2 v_2)T]_{i+(1/2)j}^{n+1} - [r(\rho_1 c_1 v_1 + \rho_2 c_2 v_2)T]_{i-(1/2)j}^{n+1}\}s_\alpha\,\Delta t$$

$$+ \{[(\rho_1 c_1 v_1 + \rho_2 c_2 v_2)T]_{ij+1/2}^{n+1} - [(\rho_1 c_1 v_1 + \rho_2 c_2 v_2)T]_{ij-1/2}^{n+1}\}s_r\,\Delta t$$

$$= \Big((r\lambda_r)_{i+(1/2)j}^{n+1}\,\frac{T_{i+1j}^{n+1} - T_{ij}^{n+1}}{\Delta r_i^e} - (r\lambda_r)_{i-(1/2)j}^{n+1}\,\frac{T_{ij}^{n+1} - T_{i-1j}^{n+1}}{\Delta r_i^w}\Big)s_\alpha\,\Delta t$$

$$+ \Big(\lambda_{\alpha ij+1/2}^{n+1}\,\frac{T_{ij+1}^{n+1} - T_{ij}^{n+1}}{\Delta\alpha} - \lambda_{\alpha ij-1/2}^{n+1}\,\frac{T_{ij}^{n+1} - T_{ij-1}^{n+1}}{\Delta\alpha}\Big)s_r\,\Delta t + q_{DTij}^{n+1}\,\Delta t + q_{adij}^{n+1}\,\Delta t,$$

where q_{DTij}^{n+1} and q_{adij}^{n+1} are terms due to the Joule–Thomson effect and the adiabatic effect:

$$q_{DTij}^{n+1} = \frac{1}{2}\Big(\varepsilon_1\rho_1 c_1\,\frac{k_1}{\mu_1} + \varepsilon_2\rho_2 c_2\,\frac{k_2}{\mu_2}\Big)_{ij}^{n+1}\Big[K_{ri-(1/2)j}\Big(\frac{P_{ij} - P_{i-1j}}{\Delta r_i^w}\Big)^2 (r_i^2 - r_{i-1/2}^2)$$

$$+ K_{ri+(1/2)j}\Big(\frac{P_{i+1j} - P_{ij}}{\Delta r_i^e}\Big)^2 (r_{i+1/2}^2 - r_i^2)\Big]^{n+1} s_\alpha$$

$$+ \Big(\varepsilon_1\rho_1 c_1\,\frac{k_1}{\mu_1} + \varepsilon_2\rho_2 c_2\,\frac{k_2}{\mu_2}\Big)_{ij}^{n+1}\Big[K_{\alpha ij-1/2}\Big(\frac{P_{ij} - P_{ij-1}}{\Delta\alpha}\Big)^2 + K_{\alpha ij+1/2}\Big(\frac{P_{ij+1} - P_{ij}}{\Delta\alpha}\Big)^2\Big]^{n+1} s_r,$$

$$q_{adij}^{n+1} = V_{ij} m_{ij}\Big(\rho_1 c_1 S_1 \eta_1 + \rho_2 c_2 S_2 \eta_2\Big)_{ij}^{n+1}\,\frac{P_{ij}^{n+1} - P_{ij}^{n}}{\Delta t}.$$

The problem was tested by comparing the known analytical solution for the thermal field due to the barothermic effect (liquid temperature variation due to the Joule–Thomson effect and the adiabatic effect in a nonstationary pressure field) during filtration of single-phase one-component oil with calculated time dependences of the temperature at the reservoir exit [2, 8]. The difference of the corresponding temperature values does not exceed 1%.

Figure 2–5 give calculation results for the following thermodynamic parameters of the phases: $c_0 = 800$ J/(kg $\cdot$ K), $c_1 = 1880$ J/(kg $\cdot$ K), $c_2 = 4200$ J/(kg $\cdot$ K), $\varepsilon_1 = 0.4$ K/MPa, $\varepsilon_2 = 0.2$ K/MPa, $\eta_1 = 0.17$ K/MPa, and $\eta_2 = 0.015$ K/MPa. The viscosities of the oil phase and water are set equal to $\mu_1 = 0.005$ MPa $\cdot$ sec and

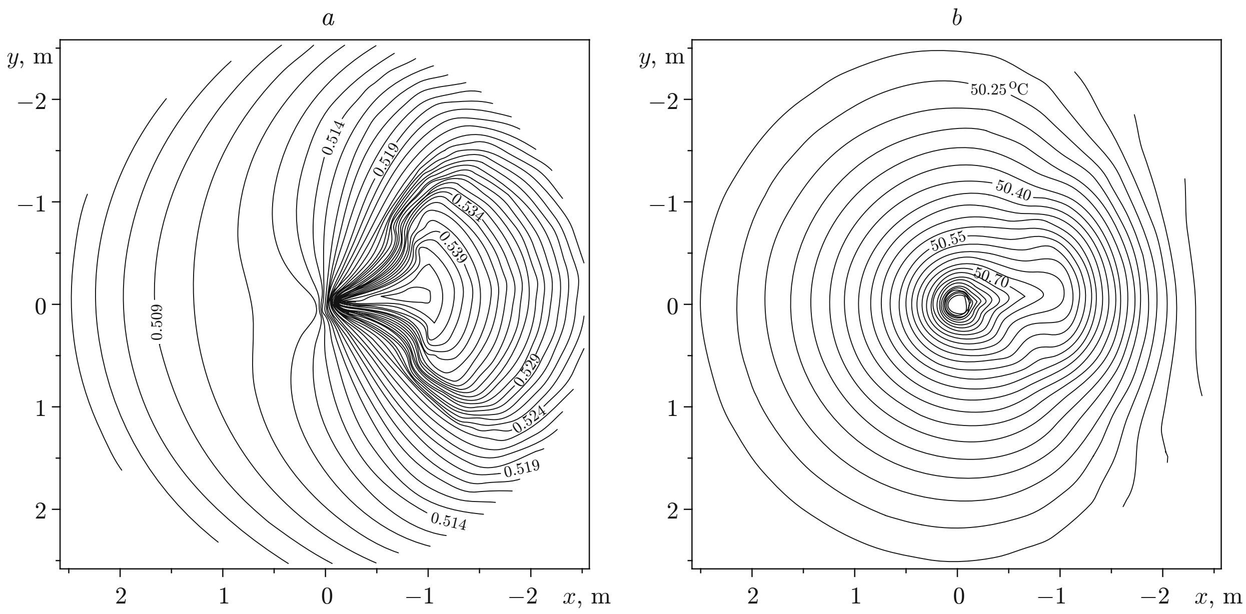

Fig. 3. Isolines of oil saturation S_1 (a) and temperature T (b) in the vicinity of the well at $k_{\text{oil}} = 0.1K$.

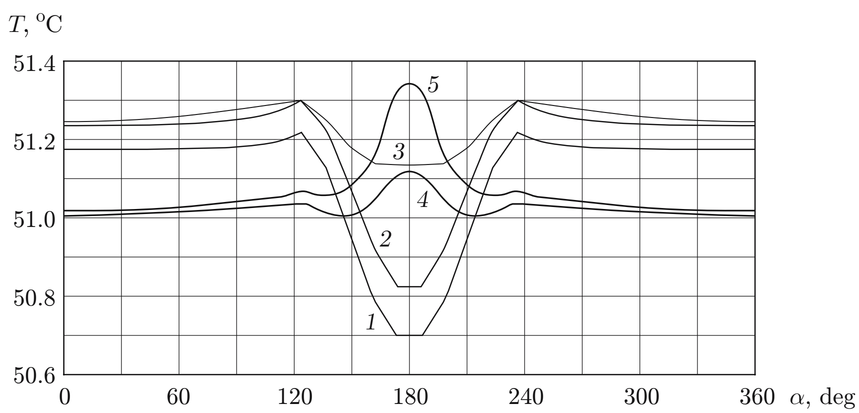

Fig. 4. Angular temperature distribution at the reservoir exit at $k_{\text{oil}} = 0.1K$ and $\alpha_{\text{oil}} = 60^\circ$ for various times: $t = 5$ (1), 10 (2), 40 (3), 60 (4), and 65 h (5).

$\mu_2 = 0.001$ MPa · sec, respectively. The reservoir pressure is $P_{\text{res}} = 10$ MPa, and the pressure on the reservoir (well) boundary is $P_{\text{w}} = 5$ MPa. The region of nonuniformity has the following dimensions: $r_{1,\text{oil}} = 1$ m, $r_{2,\text{oil}} = 2$ m, $\alpha_{\text{oil}} = 60^\circ$, $r_{\text{w}} = 0.11$ m, and $R_k = 50$ m.

Let us consider the cases where the permeability of the nonuniform region is higher and lower than the reservoir permeability. In the first case, $k_{\text{oil}} = 0.1K$, and in the second case, $k_{\text{oil}} = 10K$.

Figure 2 shows the pressure distribution in the reservoir for the cases $k_{\text{oil}} = 0.1K$ and $10K$. In the lower-permeability region, the pressure gradient increases, and in the higher-permeability region, it decreases. The pressure distribution influences the temperature field distribution in the reservoir due to the Joule–Thomson effect and the adiabatic effect. The variation in the pressure gradient in the nonuniform region leads to different temperature variations in the radial and angular directions.

Figure 3 shows isolines of oil saturation and temperatures at $k_{\text{oil}} = 0.1K$. In the lower-permeability region, there are anomalies in the oil saturation and temperatures. The angular temperature distributions for single-phase oil filtration (curves 1–3 in Fig. 4) and two-phase filtration after water breakthrough from the reservoir into the

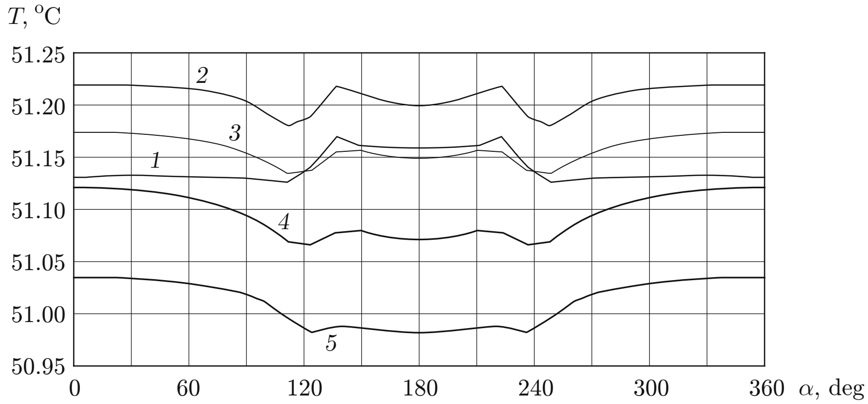

Fig. 5. Angular temperature distribution at the reservoir exit at $k_{\rm oil} = 10K$, $\alpha_{\rm oil} = 60^\circ$ for various times: $t = 5$ (1), 20 (2), 35 (3), 40 (4), and 50 h (5).

well (curves 4 and 5 in Fig. 4) are different. For single-phase filtration before water breakthrough, the region of nonuniform permeability distribution is invaded by a liquid at lower temperature and after water breakthrough, by a heated two-phase mixture of water and oil.

Thus, the angular temperature distribution shows a transition from negative to positive temperature anomalies.

For the case $k_{\rm oil} = 10K$, the angular temperature distribution in the near-well region is shown in Fig. 5. In this case, after water breakthrough, the temperature is lower in the nonuniformity region (curves 4 and 5 in Fig. 5) than in the uniform region.

The results add to the known data on the formation of temperature fields in a reservoir with nonisothermal oil and water filtration with the thermodynamic effects taken into account and can be used to interpret the results of multisensor temperature studies of wells under two-phase filtration conditions.

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